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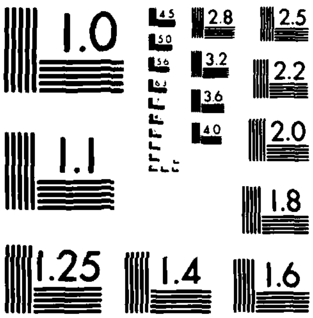

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ABSTRACT

Consider the regression model $Y_i^* = g(x_i^*) + e_i^*$ $i = 1, 2...n$. X_i^*'s are unordered design variables, g unknown function defined on [0,1]. $\{e_i^*\}$ i.i.d r.v with mean 0 and finite moment of order $p > 1$. The asymptotic behavior of estimator g_n are studied.

Key Words: Nonparametric regression, Kernel estimation, large sample property.

We consider the regression model

$$Y_i^* = g(x_i^*) + e_i^* \qquad i = 1, 2, \ldots, n \tag{1}$$

where $x_i^* \in [0,1]$ are unordered design variables. e_i^* are iid random variables with zero mean. $g(x)$ is an unknown function defined on the interval $[0,1]$. The problem is to estimate $g(x)$ through observable variables Y_i^*.

Denote ordered design variables by $x_1 \leq x_2 \leq \ldots \leq x_n$, and define $Y_i = Y_j^*$ and $e_i = e_j^*$ if $x_i = x_j^*$ and $\delta_n = \max_{1 \leq i \leq n+1} (x_i - x_{i-1})$ where $X_0 \equiv 0$, $X_{n+1} \equiv 1$. An estimator proposed by Lin and Cheng [1] is given by

$$g_n(x) = \sum_{i=1}^{n} Y_i \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz \tag{2}$$

where $\{a_n\}$ is a sequence of positive constants converging to 0. as $n \to \infty$, $k(z)$ is a kernel density satisfying:

(i) $k(z) \geq 0$;

(ii) $k(z) = 0$ for $z \notin [-L, L]$ for some positive constant L;

(iii) $\int_{-L}^{L} k(z)dz = 1$.

In fact, it is obtained by the natural estimator

$$h(x) = \begin{cases} Y_1 & x \leq x_1 \\ Y_{i+1} & x_i < x \leq x_{i+1} \quad i = 1, \ldots, n-1 \\ Y_n & x > x_n \end{cases} \tag{3}$$

after smoothing with weight function $a_n^{-1} k[(x - z)/a_n]$. Lin and Cheng [1] discussed the strong convergence of g_n and the rates of uniform convergence under a condition

$E|e^*|^p < \infty$, $p \geq 2$ among other conditions. In this note we relax the restrictions on the moments of e_i^* and improve the results of [1] and [2]. The results are given in the following theorem:

<u>Theorem</u> Assume that k(z) is bounded, $E|e^*|^p < \infty$ for $p > 1$, $\delta_n = 0(n^{-1})$, $a_n = n^{-d}$, for $0 < d < 1 - \frac{1}{p}$. If g(x) is continuous in [0,1], then for all $x \in (0.1)$

$$g_n(x) \to g(x) \qquad \text{a.s.} \tag{4}$$

Proof: Denote $\tilde{e}_i = e_i I(|e_i| \leq i^{1/p})$.

$\tilde{g}_n = \sum_{i=1}^{n} \tilde{e}_i \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz$, where I(·) is an indicator function. Then

$$\begin{aligned} & g_n(x) - g(x) \\ &= g_n(x) - \tilde{g}_n(x) + \tilde{g}_n(x) - E\tilde{g}_n(x) + E\tilde{g}_n(x) - Eg_n(x) + Eg_n(x) - g(x) \\ &\overset{\Delta}{=} I_1 + I_2 + I_3 + I_4. \end{aligned}$$

In the following we prove $I_i \xrightarrow{a.s.} 0$, i = 1, 2 and $I_j \to 0$, j = 3, 4 separately. First, when n is large enough

$$\begin{aligned} I_4 = Eg_n - g &= \sum_{i=1}^{n} g(x_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz - g(x) \\ &= \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} [g(x_i) - g(z)] a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz \\ &\quad + \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} [g(z) - g(x)] a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz \end{aligned} \tag{6}$$

By the uniform continuity of g(x) in [0.1] and the property of k(z), the first term above is less than $\varepsilon \int_0^1 \frac{1}{a_n} k\left(\frac{x - z}{a_n}\right) dz$. where ε is arbitrary positive constant. The second term in (6) is not greater than

$$\sup_{|z-x|<\delta} |g(z)-g(x)| + \frac{1}{\delta} \sup_{|t|>\frac{\delta}{a_n}} |tk(t)| \int_0^1 |g(z)|\,dz$$

$$+ \sup_x |g(x)| \int_{|u|>\frac{\delta}{a_n}} k(u)du.$$

So it is easy to see when $n \to \infty$,

$$I_4 \to 0 \tag{7}$$

by Holder and Chebyshev inequality.

$$|I_3| \le \sum_{i=1}^{n} E\{|e_i| I(|e_i| > i^{1/p})\} \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz$$

$$\le a_n^{-1}\delta_n \|k\| \sum_{i=1}^{n} [P(|e_i| > i^{1/p})]^{1-\frac{1}{p}} (E|e_i|^p)^{\frac{1}{p}}$$

$$\le a_n^{-1}\delta_n \|k\| \sum_{i=1}^{n} \frac{1}{i^{1-\frac{1}{p}}} (E|e_i|^p)^{1-\frac{1}{p}} (E|e_i|^p)^{\frac{1}{p}}$$

$$\le c\, a_n^{-1}\delta_n \sum_{i=1}^{n} \frac{1}{i^{1-\frac{1}{p}}} \le c\, a_n^{-1}\delta_n n^{\frac{1}{p}} \to 0, \qquad n \to \infty \tag{8}$$

where $\|k\| = \sup_x |k(x)|$, c a constant. Notice that if $1 < b \le 2$, we have *$e^z \le 1 + z + |z|^b$ when $z \le 1$, so if r.v $Z_i \le 1$, $EZ_i = 0$ then $E(\exp\{Z_i\}) \le \exp(E|Z_i|^b)$*. Now, take $b < p$, $d_n = \ell_n n \ell_n \ell_n n$, $Z_i = d_n(\tilde{e}_i - E\tilde{e}_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz$. By the choice of d_n and the conditions on δ_n and a_n in the theorem we know that $Z_i \le 1$ when n is large enough. Thus, we have

$$E\left(\prod_{i=1}^{n} \exp\{Z_i\}\right) = \prod_{i=1}^{n} E(\exp\{Z_i\}) \le \prod_{i=1}^{n} \exp(E|Z_i|^b)$$

$$\le \prod_{i=1}^{n} \exp\{d_n (d_n \|k\| \delta_n a_n^{-1})^{b-1} E|e_i|^b \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz\}$$

$$\le \exp\{c\, d_n (d_n \delta_n a_n^{-1})^{b-1}\}$$

For given $\varepsilon > 0$, when n is large enough,

$$P(I_2 > \varepsilon) \leq e^{-d_n \varepsilon} E(\exp\{d_n I_2\})$$

$$= e^{-d_n \varepsilon} E\left(\prod_{i=1}^{n} \exp\{Z_i\}\right) < e^{-\frac{1}{2} d_n \varepsilon}.$$

So, we have

$$\sum_n P(I_2 > \varepsilon) < \sum_n e^{\frac{1}{2} d_n \varepsilon} < \infty$$

by Borel-Cantelli lemma

$$\limsup_{n \to \infty} I_2 \leq 0 \qquad \text{a.s.}$$

Similarly, it is true that

$$\liminf_{n \to \infty} I_2 \geq 0 \qquad \text{a.s.}$$

Therefore

$$I_2 \to 0 \qquad \text{a.s.} \tag{9}$$

Since $E|\, e_i\, |^p < \infty$, we know that $\sum_n P(|\, e_i\, | > j^{\frac{1}{p}}) < \infty$. By Borel-Cantelli lemma we have $P(|\, e_i| > i^{\frac{1}{p}}, \text{ i.o}) = 0$. Hence

$$\sum_{i=1}^{\infty} e_i^2 I(|\, e_i\, | > i^{\frac{1}{p}}) < \infty \quad \text{a.s.} \tag{10}$$

Finally,

$$|\, I_1\, |^2 = |\sum_{i=1}^{n} (e_i - e_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x - z}{a_n}\right) dz|^2.$$

By Schwartz inequality,

$$|\, I_1\, |^2 \leq \sum_{i=1}^{n} \left(\int_{x_{i-1}}^{x_i} a_n^{-1}\right) k\left(\frac{x - z}{a_n}\right) dz)^2 \sum_{i=1}^{n} e_i^2 I(|\, e_i\, | > i^{\frac{1}{p}})$$

$$\leq a_n^{-1}\delta_n \, \| k \| \int_0^1 a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz \sum_{i=1}^{\infty} e_i^2 I(| e_i | > i^{\frac{1}{p}})$$

$$\leq c\, a_n^{-1}\delta_n \sum_i e_i^2 I(| e_i | > i^{\frac{1}{p}}).$$

By (10) we have

$$| I_1|^2 \to 0 \quad \text{a.s.} \tag{11}$$

From (5), (7), (8), (9), and (11), the theorem follows.

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